W #25 Fall 2008

$\frac{5}{5}$

Prove: $\sum(Y_i - \bar{Y})^2 = \sum(\hat{Y}_i - \bar{Y})^2 + \sum(Y_i - \hat{Y}_i)^2$

Proof: $\sum(Y_i - \bar{Y})^2 = \sum(Y_i + (\pm \hat{Y}_i) - \bar{Y})^2 = \sum\Big[\overset{a}{(Y_i - \hat{Y}_i)} + \overset{b}{(\hat{Y}_i - \bar{Y})}\Big]^2$

$\underline{\sum a_i b_i} = (\sum a_i) b_i$

$\sum$

$[\sum a_i] = \sum \cdot b_i$

$$= \sum\left\{(Y_i - \hat{Y}_i)^2 + (\hat{Y}_i - \bar{Y})^2 + 2(Y_i - \hat{Y}_i)(\hat{Y}_i - \bar{Y})\right\}.$$

$$= \sum(Y_i - \hat{Y}_i)^2 \checkmark + \sum(\hat{Y}_i - \bar{Y})^2 \checkmark + 2\sum(Y_i - \hat{Y}_i)(\hat{Y}_i - \bar{Y})$$

$\sum(Y_i - \hat{Y}_i)$

Need to show $\cancel{2}\sum(Y_i - \hat{Y}_i)(\hat{Y}_i - \bar{Y}) = 0$.

$$\underline{\hat{Y}_i = B_0 + B_1 X_i} \qquad \underline{B_0 = \bar{Y} - B_1 \bar{X}}$$

so $\hat{Y}_i = (\bar{Y} - B_1 \bar{X}) + B_1 X_i = \bar{Y} + B_1(X_i - \bar{X})$

so $\sum(Y_i - \hat{Y}_i)(\cancel{\bar{Y}} + B_1(X_i - \bar{X}) - \cancel{\bar{Y}})$

$$= \sum(Y_i - \hat{Y}_i) B_1 (X_i - \bar{X}) = B_1 \sum(Y_i - \hat{Y}_i) X_i - B_1 \bar{X} \sum(Y_i - \hat{Y}_i).$$

$$= B_1 \underbrace{\sum(Y_i - B_0 - B_1 X_i)(X_i)}_{=0} + B_1 \bar{X} \underbrace{\sum(Y_i - B_0 - B_1 X_i)}_{=0}$$

But recall B_0 + B_1 are our LSE + hence satisfy

$$\frac{\partial \sum(Y_i - B_0 - B_1 X_i)^2}{\partial B_0} = 0 \Rightarrow \sum(Y_i - \underbrace{B_0 - B_1 X_i}_{\hat{Y}_i}) = 0$$

$$\frac{\partial \sum(Y_i - B_0 - B_1 X_i)^2}{\partial B_1} = 0 \Rightarrow \sum(Y_i - B_0 - B_1 X_i)(X_i) = 0$$

so

$$\sum(Y_i - \bar{Y})^2 = \sum(Y_i - \hat{Y}_i)^2 + \sum(\hat{Y}_i - \bar{Y})^2.$$