NAME: Answer Key

Directions: Be sure to show all of your work. An answer alone will not receive any credit. You must show a formula or how you arrived at your answer. Partial credit will be given on all problems.

[20] 1. True/False questions.

T F [2]a. If the sample size is large (n > 30), the population is approximately Normally distributed.

T F [2]b. If the sample size is large (n > 30), the sample is approximately Normally distributed.

T F [2]c. If the sample size is large (n > 30), the population mean is approximately Normally distributed.

T F [2]d. If the sample size is large (n > 30), the sample average is approximately Normally distributed.

T F [2]e. A Margin of Errors is the half width of a confidence interval for a parameter.

T F [2]f. The estimated standard error of $\hat{p}$ is $\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$.

T F [2]g. $t_{(0.995, 12)} = -3.055$.

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A 99% confidence interval for the mean amount paid for a haircuts by MU females was calculated for all of the STA 301 students I have surveyed in the past and yielded: [$21.84, $39.76], with an average of $30.80. This CI was based on 58 female responses. Use this information to answer the T/F below.

T F [2]h. We can be fairly certain that at least ½ of all MU females spend at least $39.76 for haircuts.

T F [2]i. If I took another sample of 58 MU females, the average amount these females paid for a haircut would fall between 21.84 and 29.76 (3), 99% of the time.

T F [2]j. We can conclude the mean amount MU females spend on haircuts is likely closer to $30.80 than to $21.84.

[10] 2. In a recent *USA Today* Snapshots, it was reported that Gen Xers (ages 29-40) with millennial kids (ages 9-28) at home are more likely to have portable digital media players such as iPods. They report that 19% of 537 Gen Xers with millennial kids at home own portable digital media players, while only 10% of 602 Gen Xers without millennial kids at home own one. The implication of the article is that Gen Xers with millennial kids are more tech savy than Gen Xers without millennial kids. If we wished to prove this implication using a hypothesis test, what would be the parameter(s) and hypotheses?

5 0. p_W = proportion of GXers w/ Mill kids at home who own port. media players.
$p_{\not{W}}$ = " " " w/o " " "

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(-3) if 8

5 1. H_0: $p_W = p_{\not{W}}$ (or <) 2

2. H_A: $p_W > p_{\not{W}}$ 3

15 3. A machine is producing metal pieces that are cylindrical in shape. Nine pieces are taken and the diameters measured, yielding: 1.01, 0.97, 1.03, 1.04, 0.99, 0.98, 0.99, 1.01, and 1.03 cm. Find a 99% confidence interval for the mean diameter of pieces from this machine. What do you need to assume in order to answer the question?

2 • since not stated need to assume 9 pieces are a RS.
2 • n small & σ unknown ∴ need to assume popln Normal Then CI is $\bar{x} \pm t \cdot s/\sqrt{n}$

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4 • from calc: $n = 9$ $\bar{x} = 1.0056$ $s = 0.0246$
3 • 99% => $\alpha = 0.01$ => $t(.005; 8) = 3.250$
2 • So $1.0056 \pm 3.355 \times \underbrace{0.0246/\sqrt{9}}_{.0082}$ or 1.0056 ± 0.0275 or $[0.9781, 1.0331]$
2 • We are 99% conf that the mean diam is btwn $[0.9781, 1.0331]$ (-1)

15 4. A health spa claims that new exercise program will reduce a person's waist size by ~~w~~ 2 cm~~s~~ on the average over a 5-day period. The waist sizes of 6 men who participated in this exercise program are recorded before and after the 5-day period

Man	1	2	3	4	5	6	n	Avg	SD
Waist Size Before	90.4	95.5	98.7	115.9	104.0	85.6	6	98.35	10.7139
Waist Size After	91.7	93.9	97.4	112.8	101.3	84.0	6	96.85	9.7453
Before-After Difference	-1.3	1.6	1.3	3.1	2.7	1.6	6	1.50	1.5427

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By computing a 95% confidence interval for the mean reduction in waist size, determine whether the health spa's claim is valid. Normality is a reasonable assumption.

5 • Since samples of Before/After NOT independent [same man measured twice] use Paired Sample CI. $\bar{d} \pm t(\alpha/2; n-1)\, s_d/\sqrt{n}$

If use Indep Sample $[-11.69, 14.69]$
$df = 9.9115$ $\sqrt{\frac{s_1^2}{n_1}+\frac{s_2^2}{n_2}} = 5.9127$
$t(.025; 10) = 2.228$ $t(.025; 9) = 2.262$

3 • $\alpha = 0.05$ $t(.025; 5) = 2.571$
2 • So $1.50 \pm 2.571 \underbrace{1.5427/\sqrt{6}}_{0.6298}$ or 1.50 ± 1.6192 or $[-0.1192, 3.1192]$
2 • We are 95% conf mean reduction in waist size is btwn -0.1192 and 3.1192. (1) (1)
3 • Claim may or may not be true since μ values < 2 in CI.

15 5. A survey was done with the hope of comparing salaries of chemical plant managers employed in two areas of the country, the northern and west central regions. Independent random samples of 300 plant managers were selected for each of the two regions. These managers were asked their annual salaries. Obtain a 98% confidence interval for the difference in West Central and Northern chemical plant managers mean salaries. The results are:

Northern	West Central
$\bar{x}_1 = \$102{,}300$	$\bar{x}_2 = \$98{,}500$
$s_1 = \$5{,}700$	$s_2 = \$3{,}800$

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3 • Indep Samples so CI $(\bar{x}_1 - \bar{x}_2) \pm t(\alpha/2, df)\sqrt{\frac{s_1^2}{n_1}+\frac{s_2^2}{n_2}}$

5 • $df = \left[\frac{5700^2}{300} + \frac{3800^2}{300}\right]^2 \Big/ \left\{\frac{\left[\frac{5700^2}{300}\right]^2}{299} + \frac{\left[\frac{3800^2}{300}\right]^2}{299}\right\} = \frac{2.4471\times10^{10}}{46975611.30} = 520.9381 = 520$ (-1)

3 • $\alpha = 0.02$ $t(.01; 520) \approx z_{.01} = 2.326$
2 • $(102{,}300 - 98{,}500) \pm 2.326\sqrt{\frac{5700^2}{300}+\frac{3800^2}{300}}$ => $3800 \pm 2.326 \cdot (395.5165)$ ± 919.97
2 • We are 98% conf that Chem plant managers in Northern region make on average \$2,880.03 to \$4,719.97 more than managers in West Central.

10 6. A study is to be made to estimate the proportion of residents in a certain city and its suburbs who favor the construction of a nuclear power plant. How large a sample is needed if one wishes to be at least 99% confident that the estimate is within 0.04 of the true proportion of residents in this city and its suburbs that favor the construction of a nuclear power plant?

- $\hat{p}$ within 0.04 of $p \Rightarrow 0.04 = z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$ $\hat{p} = .5$ (-2) $\alpha = 0.01$ $z_{.005} = 2.576$ (2)
- but $\hat{p}(1-\hat{p}) \le \frac{1}{4}$ for all $\hat{p}$

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- $n = \left(\frac{2.576}{0.04}\right)^2 \hat{p}(1-\hat{p}) = 64.4^2\,\hat{p}(1-\hat{p}) \le 4147.36 \cdot \frac{1}{4} = 1036.84$
- So choose $n = 1037$ will ensure $\hat{p}$ within at most 0.04 of p with 99% conf.

(-3) $\frac{\hat{p}(1-\hat{p})}{.0002}$

$\boxed{\text{OR}}$ (5) $n = \frac{z_{\alpha/2}^2}{4 * e^2} \approx \frac{2.576^2}{4\,(0.04)^2} = 1{,}037$

15 7. Suppose we have two populations with means μ_1 and μ_2 and variances σ_1^2 and σ_2^2, respectively. Consider two large random samples of size n_1 and n_2, respectively, from these populations and let $\bar{X}_1$ and $\bar{X}_2$ be the respective sample averages.

5 a. What is the distribution of $\bar{X}_1$? What theorem did you use to answer this question?

By Central Limit Theorem (2) $\bar{X}_1 \approx \text{Normal}$ (1) $\left(\mu_{\bar{X}_1} = \mu_1\right.$ (1), $\left.\sigma^2_{\bar{X}_1} = \frac{\sigma_1^2}{n_1}\right)$ (1).

5 b. We claim that $\bar{X}_1 - \bar{X}_2$ is unbiased for $\mu_1 - \mu_2$. Prove that this is a true statement?

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unbiased means $E[\bar{X}_1 - \bar{X}_2] = \mu_1 - \mu_2$ likewise as above for $\bar{X}_2$

$E[\bar{X}_1 - \bar{X}_2] = E[\bar{X}_1] - E[\bar{X}_2] = \mu_{\bar{X}_1} - \mu_{\bar{X}_2} = \mu_1 - \mu_2$ done.

5 c. Find the standard error of $(\bar{X}_1 - \bar{X}_2)$. What did you assume?

$$se(\bar{X}_1 - \bar{X}_2) = \sqrt{var(\bar{X}_1 - \bar{X}_2)} = \sqrt{var(\bar{X}_1) + var(\bar{X}_2)} = \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

only true if RS's are independent. (2)