NAME: Answer Key

Directions: Be sure to show all of your work. An answer alone will not receive any credit. You must show a formula or how you arrived at your answer. Partial credit will be given on all problems.

35 1. Short answer questions.

10 a. According to the Roanoke *Times and World-News*, approximately 2/3 ($\hat p$) of the 1600 (n) adults polled by telephone said they think the space shuttle program is a good investment of the country. What would be the 98% margin of error for the estimated proportion of adults who think the space shuttle program is a good investment of the country?

$\alpha = .02$, $\frac{\alpha}{2} = .01$

(2) n large? $n\hat p = \frac{2}{3}1600 = 1067 > 5$ $n(1-\hat p) = 1600\left(\frac{1}{3}\right) = 533 > 5$ so yes.

10/10

$\underbrace{\text{M of E} = z_{\alpha/2}\sqrt{\frac{\hat p(1-\hat p)}{n}}}_{(2)} = z_{.01}\sqrt{\frac{2/3 \cdot 1/3}{1600}} \overset{(2)}{=} 2.326\sqrt{.00013889} = 2.326(0.0118)$

$\boxed{= 0.0274}$

← ½ CI

10 b. Given a normal random variable with mean 20 and variance 9, and a random sample of size n from the distribution, what sample size, n, is necessary in order that Pr{ 19.9 < $\bar{X}$ < 20.1 } = 0.97 ?

$.1 = z_{\alpha/2}\sqrt{\frac{9}{n}}$ (2)

10/10

RV = $N(20, 9)$ n = RS $\bar X = N(20, 9/n)$ N since popln N

$P(19.9 < \bar X < 20.1) = P\left(\frac{19.9-20}{\sqrt{9/n}} < Z < \frac{20.1-20}{\sqrt{9/n}}\right) = .97 = P(-2.170 \le Z < 2.170)$

so $\boxed{2.170 = \frac{20.1-20}{\sqrt{9}}\cdot\sqrt{n}} = 2.170 = 0.0333\sqrt{n}$ $n = \left[\frac{2.170}{.0333}\right]^2 = 65.1^2 = 4238.01$

$\boxed{= 4238}$

35/35

15 c. Here are the results from the survey the beginning of the semester to the question "Have you ever been out of the continental US?" In my STA 671 graduate class, 19 responded yes out of 24, while in all of my undergraduate sections of STA 301, 20 said yes out of 55. $\hat p_G = \frac{19}{24}$ $\hat p_u = \frac{20}{55}$

Obtain a 99% confidence interval for the difference in proportions of graduate and undergraduate students who have travelled outside of the US. (4)

n's large then $p_G - p_u$ is $\hat p_G - \hat p_u \pm z_{\alpha/2}\sqrt{\frac{\hat p_G(1-\hat p_G)}{n_G} + \frac{\hat p_u(1-\hat p_u)}{n_u}}$

Can you conclude one group of the other has been out of the US more? Explain your answer.

15/15

both n's large in (3) $n_G\hat p_G = 19$ + $n_u\hat p_u = 20$ + $n_G(1-\hat p_G) = 5 > 5$ ✓

$n_u(1-\hat p_u) = 55\left(1-\frac{20}{55}\right) = 35$ ✓

so $z_{.005} = 2.576$ $\left(\frac{19}{24} - \frac{20}{55}\right) \pm 2.576\sqrt{\frac{\frac{19}{24}\cdot\frac{5}{24}}{24} + \frac{\frac{20}{55}\cdot\frac{35}{55}}{55}} = 2.576\cdot\sqrt{.0111}$

$\pm 2.576\cdot(0.1054) = \pm 0.2715$

so $.4280 \pm 0.2715$ is $(.1565, 0.6995)$

(3) Conclude a greater proportion of grad students have been out of the country compared to undergrads b/c 0 not in CI and CI is all +.

15 3. The following measurements were recorded for the drying time, in hours, of a certain brand of latex paint:

3.4 2.5 4.8 2.9 3.6 2.8 3.3 5.6 3.7 2.8 4.4 4.0 5.2 3.0 4.8

$n = 15$

$\Sigma x = 56.8$ $\Sigma x^2 = 228.28$

μ parameter

A 99% confidence interval for the mean drying time of this brand of paint is to be calculated. **DO NOT CALCULATE THIS CI, BUT ANSWER THE QUESTIONS BELOW!**

3 a. What is your point estimate of the parameter of interest? from calc $\bar{x} = 3.7867$ (−2) not given

(−2)

4 b. What is your multiplier? $n = 15$ small so $t(\alpha/2; n-1) = t(.005; 14) = 2.977$ (1) (−2) 4 .005 (−2) 7 x

3 c. What must you assume about the population in order to use the multiplier in part b)?

Popln of drying times must be Normal

5 d. What is the standard error of you point estimate? from calc

$se(\bar{x}) = \frac{A}{\sqrt{n}} = \frac{0.9709}{\sqrt{15}} = 0.2507$ (−1)

$se = \sigma_{\bar{x}}$ (−3) $s^2 = \frac{1}{n(n-1)}\left(n\Sigma x_i^2 - (\Sigma x)^2\right)$

use $\sigma_x = .9380$ (−2) $= .9427$ (−1) $= s$.

20 4. A random sample of 64 bags of white cheddar popcorn weighed, on average, 5.23 ounces with a standard deviation of 0.24 ounces. Test the hypothesis the mean weight of cheddar popcorn is less than 5.5 ounces. Use a level of significance of 0.05. USE THE 8 STEP PROCESS FROM CLASS!

s n $\bar{x}$ μ_0 α

2 0. μ = mean wgt of cheddar popcorn

2 1. $H_0: \mu = 5.5$ (or >) backwards (−2) no μ (−2) $\bar{x}$ (−2)

2 2. $H_A: \mu < 5.5$

1 3. $\alpha = 0.05$

4 4. $n = 64$ large so $\bar{X}$ (PE of μ) $\approx$ Normal$\left(\mu_{\bar{x}} = 5.5,\ \sigma^2_{\bar{x}} = \frac{\sigma^2}{n} \doteq \frac{s^2}{n} = \frac{0.24^2}{64} = .0009\right)$ (1) (1) (1)

4 5. $\bar{x} = 5.23$ (1) p-value $= P(\bar{X} < 5.23) = P\left(Z < \frac{5.23 - 5.5}{\sqrt{.0009}} = -9\right) \approx 0$ (1) (1) (1)

2 6. Reject Ho since $p \approx 0 < .05 = \alpha$ (1) (1)

3 7. With 95% confidence we can conclude that $\mu < 5.5$ ozs.

inconsistent (−2)

15 4. A random sample of 30 airline pilots was taken. Of the 30 pilots, fifteen were under the age of 50 and fifteen were over. Each pilot's reaction time (in milliseconds) to a standard situation while flying was measured. Below is the SAS output of an analysis of these data. Use this info to answer the True/False below.

Variable	Age	N	Lower CL Mean	Mean	Upper CL Mean	Lower CL Std Dev	Std Dev	Upper CL Std Dev
Time	Old	15	521.1	694.41	867.73	150.76	225.49	417.97
Time	Young	15	302.59	431.4	560.21	112.05	167.59	310.65
Time	Diff (1-2)		62.562	263.01	463.46	147.21	198.66	297.79

15/15

T F 3a. This data should have been paired and analyzed using differences, d_i's, since there are equal numbers of Old and Young pilots. No relation btwn Y + O so indep samples

T F 3b. Since the CI's for the mean reaction times of Old and Young pilots overlap, we conclude that their mean reaction times are not different. $(521.1 \to 867.73)$ $(302.59, 560.21)$

T F 3c. The mean reaction time for Old pilots, μ_{Old}, is 694.41. $= \bar{x}_{old}$ Not μ_{old}

T F 3d. The standard error of the difference in AVG reaction times between Old and Young pilots is approximately 72.54. $\sqrt{\frac{225.49^2}{15} + \frac{167.59^2}{15}} = \sqrt{5262.1432} = 72.5406$

T F 3e. Since zero is not in the CI for the difference of the mean reaction times (Diff (1-2)), we conclude the mean reaction times of Old and Young pilots are different.

15 5. Suppose a random sample of size n is selected from a population with mean μ and variance σ^2. $X_1 \cdots X_n$ iid (μ, σ^2)

$\frac{\sigma^2}{n}$ -1 $0 = -2$

2 a. What is the variance of any X_i? $= V(x_i) = \sigma^2$

3 b. What would the E [X_i^2] be? since $V(x_i) = \sigma^2 = E(x_i^2) - \mu_{x_i}^2$ $E[x_i^2] = \sigma^2 + \mu^2$

2 c. What is the variance of $\bar{X}$? $V(\bar{x}) = \sigma^2/n$

3 d. What would the E [$\bar{X}^2$] be? $= E[\bar{x}^2] - \mu_{\bar{x}}^2 = \sigma^2/n \Rightarrow E[\bar{x}^2] = \frac{\sigma^2}{n} + \mu_{\bar{x}}^2 = \frac{\sigma^2}{n} + \mu^2$

5 e. Recall that the sample variance is $S^2 = \frac{1}{n-1}\sum_{i=1}^{n}(X_i - \bar{X})^2 = \frac{1}{n-1}\left\{\sum_{i=1}^{n}(X_i)^2 - n(\bar{X})^2\right\}$. Use all of the above facts to show that S^2 is unbiased for σ^2. $\Leftrightarrow E(S^2) = \sigma^2$ 1

$$E(S^2) = E\left[\frac{1}{n-1}\left(\sum (X_i)^2 - n(\bar{x})^2\right)\right] \overset{1}{=} \frac{1}{n-1}\left[\sum^n E(x_i^2) - n\,E(\bar{x}^2)\right]$$

$$= \frac{1}{n-1}\left[\sum^n \sigma^2 + \mu^2 - n\left[\frac{\sigma^2}{n} + \mu^2\right]\right] = \frac{1}{n-1}\left[n\sigma^2 + \cancel{n\mu^2} - \sigma^2 - \cancel{n\mu^2}\right]$$

1 1

$$= \frac{(n-1)\sigma^2}{n-1} = \sigma^2 \; //$$

1