NAME: ANSWER KEY

Directions: Be sure to show all of your work. An answer alone will not receive any credit. You must show a formula or how you arrived at your answer. Partial credit will be given on all problems.

[20] 1. True/False questions.

T (F) [4]a. If X is Binomial (n = 15, p = 0.60), Pr { 10 < X ≤ 14 } = 0.~~2168~~. 0.4027

20 T (F) [4]b. $z_{0.8508} = 1.04$.

20 (T) F [4]c. If X is Bin (n = 10, p = 0.2) and Y = 10 – X, the mean and variance of Y are 8 and 1.6, respectively.

$\mu_X = 10(.2) = 2 \quad \sigma_X^2 = 10(.2)(.8) = 2(.8) = 1.6$ so $\mu_Y = 10-2 = 8$ + Var = 1.6

T (F) [4]d. A probability density function must sum to one.

(T) F [4]e. Let X be a discrete random variable with $S_X = \{-1, 0, 1, 2, 3, 4\}$ and probability function $f_X(x) = \dfrac{(x-1)^2}{19}$. The probability that X is at least zero is 15/19.

$P(X \geq 0) = 1 - P(X<0) = 1 - P(X=-1) = 1 - \frac{(-1-1)^2}{19} = 1 - \frac{(-2)^2}{19} = 1 - \frac{4}{19} = \frac{15}{19}$

[15] 2. A nationwide survey of seniors by the University of Michigan reveals that almost 70% disapprove of daily pot smoking, according to a report by Parade. Twelve seniors are selected at random and asked their opinion.

[8] a. What is the relevant random variable for this problem and what is its distribution?

8/8

1 S = # of seniors surveyed who disapprove of daily pot smoking

S = Bin (n = 12, p = 0.70) b/c 1 (1) n = 12 trials
1 (2) S/F = Disap of daily pot smoking / Approve
1 (3) P(S) = 0.70 constant
1 (4) indep trials b/c random selection

15/15

[7] b. Find the probability that the number who disapprove of smoking pot daily is not less than 8.

7/7

$P(S \text{ not less than } 8) = P(S \overset{\text{NOT}}{<} 8) = P(\underbrace{S \geq 8}_{2})$

$\underbrace{1 - P(S<8) = 1 - P(S \leq 7)}_{2} = 1 - \text{binomcdf}(12, .70, 7)$

$= 1 - \underbrace{0.2763}_{1} = \underbrace{0.7237}_{1}$

15 3. A lawyer commutes daily from his suburban home to his midtown office. The average time for a one-way trip is 24 minutes with a standard deviation of 2.8 minutes. Assume the distribution of trip times to be normally distributed.

5 a. What is the relevant random variable for this problem and what is its distribution?

$T =$ commute time from home to office $= \sim \text{Normal}(\mu = 24 \text{ min}, \sigma^2 = 2.8^2)$
2 1 1 1

15/15

10 b. If the office opens at 9:00 AM, what time must he leave to insure that he is late no more than 5% of the time?

$P(T > t) \le 0.05 \Rightarrow P\left(Z > \frac{t-24}{2.8}\right) = 0.05 \Rightarrow P\left(Z < \frac{t-24}{2.8}\right) = .95$
2 2
$= 1.645$ 2

so $t = 24 + 1.645(2.8) = 28.6060$ 2

so 0.05 of commutes take longer than 28.61 mins so to insure not late needs to leave at least 28.6060 mins before 9:00am. 2

so leave @ 8:31:39
8:31:23.64

20 4. Short answer questions.

6 a. If X and Y are two random variables with means $\mu_X = 0$ and $\mu_Y = -3$ and variances $\sigma^2_X = 4$ and $\sigma^2_Y = 3$. If X and Y are independent, find the mean and variance of the random variable $W = 4X - 2Y + 30$.

$W = 4X - 2Y + 30$

3 $E(W) = 4E(X) - 2E(Y) + 30 = 4(0) - 2(-3) + 30 = 36$ −(−2)

3 $V(W) = V(4X - 2Y + 30) = V(4X - 2Y) = 4^2 V(X) + (-2)^2 V(Y)$
$= 16(4) + 4(3) = 64 + 12 = \underline{76}$

7 b. Let X be a continuous RV with pdf given by: $f_X(x) = 6x(1-x)$, for $0 < x < 1$. Find the probability that X exceeds ¼.

$P(X > \frac{1}{4}) = 1 - P(X < \frac{1}{4}) = 1 - \int_0^{1/4} 6x(1-x)\,dx = 1 - \int_0^{1/4} 6x - 6x^2\,dx$ 2

20/20

$= 1 - \left[\frac{6x^2}{2}\Big|_0^{1/4} - \frac{6x^3}{3}\Big|_0^{1/4}\right] = 1 - \left[3\left(\frac{1}{4}\right)^2 - 2\left(\frac{1}{4}\right)^3\right] = 1 - \left[\frac{3}{16} - \frac{2}{64}\right]$
2 2

$= 1 - \frac{3}{16} + \frac{1}{32} = 27/32 = 0.8438$ 1

7 c. Reaction times to a stimulus, T, is a continuous RV with pdf given by $f_T(t) = \frac{3}{2} t^{-2}$, for $1 < t < 3$. Let $Y = \sqrt{T}$. Find the mean of Y. 2

$E(Y) = E(\sqrt{T}) = \int_1^3 \sqrt{t}\left[\frac{3}{2}t^{-2}\right]dt = \int_1^3 \frac{3}{2} t^{1/2} t^{-2}\,dt = \int_1^3 \frac{3}{2} t^{-3/2}\,dt$ (2)

$= \frac{3}{2} \frac{t^{-1/2}}{-1/2}\Big|_1^3 = -3\left[3^{-1/2} - 1^{-1/2}\right] = -3\left[\frac{1}{\sqrt{3}} - 1\right]$
2

$= 3 - \frac{3}{\sqrt{3}} = 3 - \sqrt{3} = 1.2679$ 1

−3 $E(Y) = \sqrt{E(T)}$

20 5. The "1, 2, 3" of a die is loaded so that outcomes 1, 2, and 3 (all with the same probability) are three times as likely as 4, 5, and 6 (all with the same probability). This die is rolled twice; let X be the sum of the number of dots on the top face of the two rolls. Find $f_X(x)$.

• D = outcome on one roll

7

d	1	2	3	4	5	6
$f_D(d)$	$3p$	$3p$	$3p$	p	p	p
	$3/12$	$3/12$	$3/12$	$1/12$	$1/12$	$1/12$

$\Sigma = 1 = 12p \Rightarrow p = 1/12$

• X = total (2) 1

7 outcomes:

x	2	3	4	5	6	7	8	9	10	11	12
	(11)9	(12)9	(13)9	(14)3	(15)3	(16)3	(26)3	(36)3	(46)1	(56)1	(6,6)1
		(21)9	(22)9	(23)9	(24)3	(25)3	(35)3	(45)1	(55)1	(65)1	
			(31)9	(32)9	(33)9	(34)3	(44)1	(54)1	(64)1		
				(41)3	(42)3	(43)3	(53)3	(63)3			
					(51)3	(52)3	(61)3				
						(61)3					

numerator of prob

since rolls indep and outcomes M.E. $P((i,j)) = p(i)\cdot p(j) =$ $9/144$ $(i,j) \le 3$; $1/144$ $(i,j) \ge 4$; $3/144$ mix of ≤ 3 + ≥ 4

7

x	2	3	4	5	6	7	8	9	10	11	12
$f_X(x)$	$\frac{9}{144}$	$\frac{18}{144}$	$\frac{27}{144}$	$\frac{24}{144}$	$\frac{21}{144}$	$\frac{18}{144}$	$\frac{13}{144}$	$\frac{8}{144}$	$\frac{3}{144}$	$\frac{2}{144}$	$\frac{1}{144}$
	$\frac{1}{16}$	$\frac{1}{8}$	$\frac{3}{16}$	$\frac{1}{6}$	$\frac{7}{48}$	$\frac{1}{8}$	$\frac{13}{144}$	$\frac{1}{18}$	$\frac{1}{48}$	$\frac{1}{72}$	$\frac{1}{144}$

$\Sigma = 1$ ✓

$\frac{20}{20}$

10 6. Smaller regional, commuter airlines (ones with smaller planes) overbook their flights to insure full planes and hence maximize profits. One such company, that uses planes with only 15 seats exclusively, posts a loss on every flight that is less than 20% capacity. If this company knows that 10% of passengers that make reservations fail to show up for flights, what percent of their flights lose money if they overbook by 20%?

15 seats loss if # passengers < 20% · 15 = 3 2.

10% fail to show so 90% show

overbook by 20% $\Rightarrow$ sell $15(1+.20) = 18$ seats 1

Let X = # passengers who make reservations & show up out of 18.

$\frac{10}{10}$

5 $X = \text{Bin}(n = 18, p = 0.90)$ b/c

3
- $n = 18$ trials
- S/F = show up / Fail to show up
- $P(S) = 0.90$
- assume indep.

% flights that lose \$ $= P(X < 3)$

$X < 12 = 0.0012$

$= P(X \le 2) = 0.000\ldots = 1.2556\ 10^{-14}$.