NAME: Answer Key

Directions: Be sure to show all of your work. An answer alone will not receive any credit. You must show a formula or how you arrived at your answer. Partial credit will be given on all problems.

15 1. True/False questions.

$P(Z \geq z_{0.0006}) = 0.0006 \quad z_{0.0006} = 2.51$

T (F) 3a. $z_{0.006} = -2.51$.

15/15

$= P(X \leq 10) - P(X \leq 2) = 0.9939 - 0.0600 = .9339 = .934$

(T) F 3b. If X is Bin(18, 0.30), then $\Pr(3 \leq X \leq 10) = 0.934$.

(T) F 3c. If X is $\chi^2_{(13)}$ =, then $\Pr(5.009 \leq X \leq 24.736) = 0.950$. $P(\chi^2_{13} > 5.009) - P(\chi^2_{13} > 24.736) = 0.975 - 0.025 = .950$

(T) F 3d. If X is $t_{(23)}$, then $\Pr(-0.858 \leq X \leq 0.858) = 0.600$. ← TRUE

$P(T_{23} > .858) = 0.20 \cdot P(T_{23} < -.858)$ so

15 12 9 6 3 0

(T) F 3e. If X and Y are independent random variables with variances $\sigma^2_X = 5$ and $\sigma^2_Y = 10$, then the variance of 3X + 4Y = 205 = variance of 3X – 4Y.

$V(3X+4Y) = 3^2\sigma_X + 4^2\sigma_Y = 9(5) + 16(10) = 205 = V(3X-4Y) = 3^2\sigma_X + (-4)^2\sigma_Y = 9(5)+16(10) = 205$

10 2. A traffic control engineer reports that 75% of the vehicles passing through a checkpoint are from within the state. Let X be the number of out-of-state cars among the next 9 passing the checkpoint.

5 a. What kind of random variable is X? Justify your choice! Bin Exp?

-4 discrete

① n = 9 trials = cars
② car is an out-of-state ≡ S/F
③ P(S) = P(out-of-state) = 0.25
④ cars are indep with respect to state so reasonable assumption

10/10

so $X = \overbrace{\text{Bin}(n=9, p=0.25)}^{①}$.

5 b. What is the probability that $\boxed{\text{fewer than}}$ 4 of the next 9 vehicles are from out of state? -2

$P(X<4) = P(X \leq 3) = 0.8343$ from Text Bin Table.

-1 ↓ binomcdf(9, .75, 3) = 0.00999

cdf(9, .25, 4) -1 or binomcdf(9, 0.25, 3) = 0.8343 = 0.834274292

if $p(2) = \frac{365}{365} \cdot \frac{1}{365} \cdot \frac{364}{365} \cdots \frac{356}{365} \times \binom{10}{2}$ etc.

10 3. In a class of 10 students, what is the probability that at least two students have the same birthday?

P(at least two share) = 1 − P(none share) = 1 − P(10 different Bdays) -2 if 2, 3, 4 but no order!

-2

= 1 − P(BD of Student$_1$ ∩ BD of Student$_2$ ∩ … ∩ BD of Student$_{10}$ all different)

10/10

-2

= 1 − P(BDS$_1$) · P(BDS$_2$) ⋯ P(BDS$_{10}$) since students in class are indep

355 -2

$= 1 - \left[\frac{365}{365} * \frac{364}{365} * \frac{363}{365} \times \frac{362}{365} \cdot \frac{361}{365} \cdot \frac{360}{365} \cdot \frac{359}{365} \cdot \frac{358}{365} \cdot \frac{357}{365} \cdot \frac{356}{365}\right.$

first student has any B.Day; diff day from S$_1$; diff from S$_1$ + S$_2$

-7 Bin

$= \frac{1.0154 \times 10^{23}}{365^9} = \frac{1.0154 \times 10^{23}}{1.1498 \times 10^{23}} = 0.8831$ so $1 - .8831 = .1169$

20 4. In a certain federal prison it is known that 2/3 of the inmates are under 25 years of age. It is also known that 3/5 of the inmates are males and that 5/8 of the inmates are female or 25 years of age or older.

$P(<25) = 2/3 \quad P(\geq 25) = 1/3 \quad P(M) = 3/5 \quad P(F) = 2/5 \quad P(F \text{ or } \geq 25) = 5/8$

10 a. What is the probability that a prisoner selected at random from this prison is female and at least 25 years old?

$P(F \text{ and } \geq 25) = ?$

$P(F \text{ or } \geq 25) = 5/8 = P(F) + P(\geq 25) - P(F \text{ and } \geq 25)$

$= 2/5 + 1/3 - P(F \text{ and } \geq 25)$

so $P(F \text{ and } \geq 25) = \frac{2}{5} + \frac{1}{3} - 5/8$

$= 13/120$

so $P(F \text{ and } \geq 25) = \frac{13}{120} = 0.1083$

20/20

(-4) if indep

10 b. Given that we get a female, what is the probability that prisoner selected is 25 or older?

$P(\geq 25 \mid F) = \frac{P(\geq 25 \text{ and } F)}{P(F)} = \frac{13/120}{2/5} = \frac{13}{120} \cdot \frac{5}{2} = \frac{13}{24 \cdot 2} = \frac{13}{48}$

4

-6 $P(A \cap B) = \ldots$

$= 0.2708$

20 5. Impurities in the batch of final product of a chemical process often reflect a serious problem. From considerable plant data gathered, it is known that the proportion, Y, of impurities in a batch has a density function given by:

$$f_Y(y) = 10\,(1 - y)^9 \quad 0 < y < 1.$$

10 a. A batch is considered not sellable, and hence not acceptable, if the percentage of impurities exceeds 60%. With the current quality of the process, that is the percentage of batches that are not acceptable?

$\int_1^{.60}$

$P(\text{not acceptable}) = P(Y > 0.60) = \int_{.60}^{1} 10(1-y)^9\,dy = \frac{10(1-y)^{10}}{-10}\Big|_{.60}^{1}$ (-2)

$\frac{10(1-1)^{10}}{-10} - \frac{10(1-.6)^{10}}{-10} = 0 + (1-.6)^{10} = .4^{10} = 1.04864 \times 10^{-4}$

$= 0.000104864$

20/20

so 0.000104864 or 0.0105%

10 b. Roughly 10,000 batches are produced each week. What is the mean and standard deviation of the number of unacceptable batches produced each week?

(-5) $E(Y)$ $\int_0^1 y \ldots$

X = # unacceptable produced per week out of 10,000

Assuming Bin(n = 10,000, p = 0.000104864) since n = 10,000; S/F ≡ U/A; $P(S) = P(U) = .000104864$ assume indep batches

2 1 1

Then $\mu_X = np = 10{,}000(.000104864) = \boxed{1.0486}$ 3

$\sigma_X = \sqrt{np(1-p)} = \sqrt{1.0486(1 - .000104864)} = \sqrt{1.0485} = \boxed{1.0239}$ 3

$E(Y)$ (-6)
$V(Y)$

15 6. At a carnival, you get to play a game where you choose from among three six-sided dice then the carnival huckster gets to choose a die. You both roll your die and the winner is the player with the highest number on their die. You choose the crimson die whose faces are 13, 13, 13, 1, 1, 1 and assuming the die is fair you quickly deduce the die has an expected value of 7 with a standard deviation of 6. The huckster chooses the yellow die whose faces are 14, 11, 11, 2, 2, 2. If the huckster's die is also fair, what is the mean and standard deviation of his/her die?

(-8) $\sigma = 6$

Yellow die $S_Y = \{14, 11, 11, 2, 2, 2\}$ prob function = 1/6 b/c fair

15/15

(5) Then $E[\text{Yellow die}] = \sum y \cdot f_Y(y) = 14\frac{1}{6} + 11\frac{1}{6} + 11\frac{1}{6} + 2\frac{1}{6} + 2\cdot\frac{1}{6} \cdot 2\frac{1}{6}$

$= \frac{1}{6}(14+11+11+2+2+2) = \frac{42}{6} = 7 = \mu_{yellow}$

0 (-3) $\sum(x-\mu)/6$ var not sd = (-2) $\div(n-1)$ (-3)

var Yellow $= E[Y^2] - \mu_Y^2$

(5) $\sum y^2 f_Y(y) = 14^2\frac{1}{6} + 11^2\frac{1}{6} + 11^2\frac{1}{6} + 2^2\frac{1}{6} + 2^2\frac{1}{6} + 2^2\frac{1}{6}$

$= \frac{1}{6}(196 + 121 + 121 + 4 + 4 + 4) = \frac{1}{6} 450 = 75$

(5) so $V(Y) = 75 - 7^2 = 26$ so $\underbrace{sd\,Y = \sigma_Y = \sqrt{26} = 5.0990}_{(2)}$

(-3) if $V = 75$

10 7. A company produces an item that is given a quality score, X, by a team of quality control specialists (where a lower score is better). The distribution of these quality score measurements for all of the items produced by the company is approximately Normal with a mean of 5 and a variance of 20. Any item can be returned for any reason and any returned item must be inspected, refurbished, if necessary, and shipped back to the customer, all at the expense of the company. The cost of this process is a function of the quality score of the product as follows: Cost (in dollars) = C = $10X^2 - 5X + 75$. What is the mean inspection, refurbishing, and reshipping expense to the company?

$\mu_C = E(C) = E(10X^2 - 5X + 75) = 10E[X^2] - 5E[X] + 75$ (4)

now $E[X] = \text{mean of } X = \mu_X = 5$ (1)

10/10

(3) $E[X^2]$ from $\text{Var}(X) = E[X^2] - \mu_X^2 = 20 = E[X^2] - 5^2$ so $E[X^2]$

$= 20 + 25 = 45$

so $\mu_C = E[C] = 10[45] - 5[5] + 75$ (2)

(-5) if

$10\ 5^2 - 5(5) + 75$

$= 450 - 25 + 75$

$= \$500$

(-7) if $\int x(10x^2 \ldots$