NAME: Answer Key

Directions: Be sure to show all of your work. An answer alone will not receive any credit. You must show a formula or how you arrived at your answer. Partial credit will be given on all problems.

15 1. True/False questions.

$P(Z \geq z_{0.0006}) = 0.0006$ $z_{0.0006} = 2.51$

T (F) 3 a. $z_{0.006} = -2.51$.

15/15

(T) F 3 b. If X is Bin(18, 0.30), then $\Pr(3 \leq X \leq 10) = 0.934$. $= P(X \leq 10) - P(X \leq 2) = 0.9939 - 0.0600 = .9339 = .934$

(T) F 3 c. If X is $\chi^2_{(13)}$ =, then $\Pr(5.009 \leq X \leq 24.736) = 0.950$. $P(\chi^2_{13} > 5.009) - P(\chi^2_{13} > 24.736) = 0.975 - 0.025 = .950$

(T) F 3 d. If X is $t_{(23)}$, then $\Pr(-0.858 \leq X \leq 0.858) = 0.600$. ← TRUE $P(T_{23} > .858) = 0.20 \cdot P(T_{23} < -.858)$ so

(T) F 3 e. If X and Y are independent random variables with variances $\sigma^2_X = 5$ and $\sigma^2_Y = 10$, then the variance of 3X + 4Y = 205 = variance of 3X – 4Y.

$V(3X+4Y) = 3^2\sigma_X + 4^2\sigma_Y = 9(5)+16(10) = 205 = V(3X-4Y) = 3^2\sigma_X + (-4)^2\sigma_Y = 9(5)+16(10) = 205$

15 12 9 6 3 0

10 2. A traffic control engineer reports that 75% of the vehicles passing through a checkpoint are from within the state. Let X be the number of out-of-state cars among the next 9 passing the checkpoint.

5 a. What kind of random variable is X? Justify your choice! Bin Exp?

(-4) discrete

① n = 9 trials = cars
② car is an out-of-state = S/F
③ P(S) = P(out-of-state) = 0.25
④ cars are indep with respect to state so reasonable assumption

10/10

so $X = \underbrace{Bin(n=9,\ p=0.25)}_{①}$.

5 b. What is the probability that $\boxed{\text{fewer than}}$ (-2) 4 of the next 9 vehicles are from out of state?

$P(X<4) = P(X \leq 3) = 0.8343$ from Text Bin Table.

cdf(9, .25, 4) (-1) or binomcdf(9, 0.25, 3) = 0.8343 = 0.834274292

(-1) binomcdf(9, .75, 3) = 0.00999

if $p(2) = \frac{365}{365} \cdot \frac{1}{365} \cdot \frac{364}{365} \cdots \frac{356}{365} \times \binom{10}{2}$ etc.

10 3. In a class of 10 students, what is the probability that at least two students have the same birthday?

-2 if 2, 3, 4 but! no order

P(at least two share) = 1 – P(none share) = 1 – P(10 different Bdays) (2)

10/10

= 1 – P(BD of Student$_1$ ∩ BD of Student$_2$ ∩ … ∩ BD of Student$_{10}$ all different)

= 1 – P(BDS$_1$) · P(BDS$_2$) ⋯ P(BDS$_{10}$) since students in class are indep (-2)

$= 1 - \left[\frac{365}{365} * \frac{364}{365} * \frac{363}{365} \times \frac{362}{365} \cdot \frac{361}{365} \cdot \frac{360}{365} \cdot \frac{359}{365} \cdot \frac{358}{365} \cdot \frac{357}{365} \cdot \frac{356}{365}\right.$ 355 (-2)

first student has any B.Day; diff day from S$_1$; diff from S$_1$ + S$_2$

(-7 Bin)

$= \frac{1.0154 \times 10^{23}}{365^9} = \frac{1.0154 \times 10^{23}}{1.1498 \times 10^{23}} = 0.8831$ so $1 - .8831 = .1169$

20 4. In a certain federal prison it is known that 2/3 of the inmates are under 25 years of age. It is also known that 3/5 of the inmates are males and that 5/8 of the inmates are female or 25 years of age or older.

$P(<25) = 2/3 \quad P(\geq 25) = 1/3 \quad P(M) = 3/5 \quad P(F) = 2/5 \quad P(F \text{ or } \geq 25) = 5/8$

10 a. What is the probability that a prisoner selected at random from this prison is female and at least 25 years old?

$P(F \text{ and } \geq 25) = ?$

$P(F \text{ or } \geq 25) = 5/8 = P(F) + P(\geq 25) - P(F \text{ and } \geq 25)$

$= 2/5 + 1/3 - P(F \text{ and } \geq 25)$

so $P(F \text{ and } \geq 25) = \frac{2}{5} + \frac{1}{3} - 5/8$

$= 13/120$

so $P(F \text{ and } \geq 25) = \frac{13}{120} = 0.1083$

20/20

(-4) if indep

10 b. Given that we get a female, what is the probability that prisoner selected is 25 or older?

$P(\geq 25 | F) = \frac{P(\geq 25 \text{ and } F)}{P(F)} = \frac{13/120}{2/5} = \frac{13}{120} \cdot \frac{5}{2} = \frac{13}{24 \cdot 2} = \frac{13}{48}$

4

$= 0.2708$

-6 $P(A \cap B) = \ldots$

20 5. Impurities in the batch of final product of a chemical process often reflect a serious problem. From considerable plant data gathered, it is known that the proportion, Y, of impurities in a batch has a density function given by:

$$f_Y(y) = 10(1-y)^9 \quad 0 < y < 1.$$

10 a. A batch is considered not sellable, and hence not acceptable, if the percentage of impurities exceeds 60%. With the current quality of the process, that is the percentage of batches that are not acceptable?

$\int_1^{.60}$

$P(\text{not acceptable}) = P(Y > 0.60) = \int_{.60}^{1} 10(1-y)^9\, dy = \left. \frac{10(1-y)^{10}}{-10} \right|_{.60}^{1}$

-2

$\frac{10(1-1)^{10}}{-10} - \frac{10(1-.6)^{10}}{-10} = 0 + (1-.6)^{10} = .4^{10} = 1.04864 \times 10^{-4}$

$= 0.000104864$

20/20

so 0.000104864 or 0.0105%

(-5) $E(Y)$

$\int_0^1 y \ldots$

10 b. Roughly 10,000 batches are produced each week. What is the mean and standard deviation of the number of unacceptable batches produced each week?

X = # unacceptable produced per week out of 10,000

Assuming Bin($n = 10{,}000$, $p = 0.000104864$) since $n = 10{,}000$; S/F = U/A; $P(S) = P(U) = .000104864$ assume indep batches

2 1

Then $\mu_X = np = 10{,}000(.000104864) = \boxed{1.0486}$ 3

$\sigma_X = \sqrt{np(1-p)} = \sqrt{1.0486(1-.000104864)} = \sqrt{1.0485} = \boxed{1.0239}$ 3

$E(Y)$ (-6)

$V(Y)$